

**School on Numerical Methods**  
**IMCA, Lima, Peru – 1-10 July 2015**  
**Course on Compatible Discretizations (8h)**

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Compatible discretizations aim at reproducing at the discrete level some of the structure properties of the continuous problem (local conservation, invariants, and so on). We are going to present these techniques on various model problems: diffusive PDEs, advection-diffusion, linear elasticity and incompressible flow. The outline is as follows, each block lasting approx. 2h:

- Compatible discretization of lowest-order for diffusive PDEs: notion of primal and dual meshes, discrete differential operators, Hodge operator, vertex-based and cell-based schemes, see [1];
- Compatible discretization of lowest-order for advection-diffusion: notion of contraction operator and upwinding, see [2];
- Hybrid high-order schemes for diffusive PDEs: notion of potential reconstruction, stabilization, see [3];
- Hybrid high-order schemes for solid and fluid mechanics: robustness for quasi-incompressible linear elasticity and for flows driven by large irrotational forces, see [4].

Reading the basic parts of the textbooks [5, 6] can be useful to prepare for the course.

## References

- [1] J. Bonelle and A. Ern. Analysis of compatible discrete operator schemes for elliptic problems on polyhedral meshes. *ESAIM Math. Model. Numer. Anal.*, 48(2):553–581, 2014.
- [2] P. Cantin and A. Ern. Vertex-based Compatible Discrete Operator Schemes on polyhedral meshes for advection-diffusion equations. <http://hal.archives-ouvertes.fr/hal-01141290>, 2015.
- [3] D. A. Di Pietro, A. Ern, and S. Lemaire. An arbitrary-order and compact-stencil discretization of diffusion on general meshes based on local reconstruction operators. *Comput. Meth. Appl. Math.*, 14(4):461–472, 2014.
- [4] D. A. Di Pietro and A. Ern. A hybrid high-order locking-free method for linear elasticity on general meshes. *Comput. Meth. Appl. Mech. Engrg.*, 283:1–21, 2015.
- [5] A. Ern and J.-L. Guermond. *Theory and practice of finite elements*, volume 159 of *Applied Mathematical Sciences*. Springer-Verlag, New York, 2004.
- [6] D. Di Pietro and A. Ern. *Mathematical aspects of discontinuous Galerkin methods*, volume 69 of *Mathématiques & Applications (Berlin) [Mathematics & Applications]*. Springer, Heidelberg, 2012.