LIST OF ABSTRACTS

(VII IMCA SCHOOL)

F. Cukierman

INTRODUCTION TO REPRESENTATIONS OF COMPLEX SEMISIMPLE LIE ALGEBRAS

Lecture 1

Definitions and first examples of groups and linear representations. Finite groups, symmetric groups, Young diagrams and Weyl's construction.

Lecture 2

Representations of $\mathfrak{sl}(\mathbb{C})$. Nilpotent, soluble and semi-simple Lie algebras.

Lecture 3

Weights, roots, Cartan decomposition. Classification of simple Lie algebras. Classification of linear representations.

References:

- W. Fulton and J. Harris: Representation theory, a first course.
- J. Humphreys: Introduction to Lie algebras and representation theory.

L. das Dores

RATIONAL CURVES ON SYMMETRIC POWERS OF SURFACES

Let X be a nonsingular projective surface over an algebraically closed field. We say the group $A_0(X)$ of 0-cycles modulo rational equivalence of degree 0 is finite dimensional if for sufficiently large integer d, all zero cycles of a given degree $d' > d$ are rationally equivalent to each other. In other words for every pair of zero cycles of degree d' there exist rational curves passing to the corresponding points in some symmetric power of X (up to summing an effective 0-cycle). The structure of $A_0(X)$ is closely related to the "asymptotic" behaviour of rational curves on the symmetric powers of the X. In this talk, I will use global properties of the scheme of morphisms from the projective line to symmetric powers of the surface to deduce some properties of the corresponding rational curves.

R. Gonzales

EQUIVARIANT CHOW GROUPS, LOCALIZATION AND APPLICATIONS

Part 1: Algebraic rational cells and equivariant Chow groups

We discuss a notion of algebraic rational cell with applications to intersection theory on singular varieties with torus action. Based on this, we study $\mathbb{Q}$-filtrable varieties, i.e. algebraic varieties where a torus acts with isolated fixed points, such that the associated Białynicki-Birula decomposition consists of algebraic rational cells. We show that the rational equivariant Chow group of any, possibly singular, $\mathbb{Q}$-filtrable variety is freely generated by the classes of the cell closures. As an example, we apply this result to certain spherical varieties (varieties acted upon by a connected reductive group such that a Borel subgroup acts with a dense orbit). Some parallels with the corresponding situation in equivariant cohomology and K-theory will be given.

Part 2: Localization in equivariant operational Chow groups and applications

For a complete nonsingular variety with a torus action, the localization principle asserts that one can read-off the equivariant K-theory of the variety from that of fixed point subscheme, modulo certain relations given by the fixed loci of codimension-one subtori. For singular varieties, however, such method quite often does not apply. Our goal is to show that in the setting of equivariant operational theories there is a version of the localization principle that works perfectly well for both singular and nonsingular varieties. For instance, if X is any complete variety where a torus acts with finitely many fixed points and invariant curves, then the equivariant operational Chow cohomology of X is a ring of piecewise exponential functions (a version of GKM theory). The case of equivariant projective embeddings of reductive groups is discussed in some detail.

S. Gorchinskiy

Chow motives and their applications

Lecture 1: Decompositions of Chow motives

We will present a rather general technique to express Chow motives of varieties of high dimension in terms of Chow motives of varieties of smaller dimension. In particular, we will show that a threefold with representable zero-cycles is expressed in terms of Chow motives of curves, which is a common result with V. Guletskii.

Lecture 2: Transcendence of zero-cycles and generation of modules

We will give an example of a rank r projective module over a complex algebra of dimension d which is not generated by $d+r$ elements. The construction is based on transcendence degree for elements in the Chow group of zero-cycles introduced in a common paper with V. Guletskii.

Lecture 3: Chow motives of Lefschetz type

We will discuss general criteria for Chow motives to be direct sums of powers of the Lefschetz motive. Using the Merkurjev-Suslin theorem, we will give a special criterion for Chow motives of varieties of dimension at most three. As an application, we will show the existence of phantom categories, which is a common result with D. Orlov.

V. Guletskii

Motivic obstruction to rationality in dimension 4

Lecture 1: Integral (in)decomposability of transcendental motives of surfaces over a field

The purpose of the first lecture will be to recall the refined Chow-Künneth decomposition (after Kahn-Murre-Pedrini, 2007) of the motive $M(X)$ of a smooth projective surface X over a field, and explain in detail the construction of the transcendental part of $M^2(X)$. I will also introduce the notion of integral decomposability and indecomposability of the transcendental motive $M^2_{\mathrm{tr}}(X)$, and show examples of surfaces whose transcendental motives are integrally indecomposable. Then I will prove a reduction theorem saying that if the transcendental motive of any smooth projective surface over complex numbers is integrally indecomposable, then a very general cubic hypersurface in $\mathbb{P}^5$ is not rational (the latter is a longstanding conjecture in birational geometry). Finally, I will state an expectation (a motivic version of Kulikov's Hodge-theoretical conjecture), saying that the transcendental motive of a smooth projective surface over a field of characteristic 0 is integrally indecomposable, and explain the intuition behind.

Lecture 2: The transcendental motive of the Fermat sextic in $\mathbb{P}^3$

The second lecture will be totally devoted to proving that the transcendental motive of the Fermat sextic in $\mathbb{P}^3$ is integrally indecomposable. The case is important, as the transcendental Hodge structure of this surface is known to be integrally decomposable (Auel, Boehning and Graf von Bothmer, 2013), which makes Kulikov's conjecture to be false. The integral indecomposability of the transcendental motive of the Fermat sextic shows that the motivic integral indecomposability of $M^2_{\mathrm{tr}}(X)$ is possibly the right view of the cubic fourfold irrationality conjecture. The second lecture will contain a lot of explicit motivic calculations arising from the Shioda-Katzura inductive structure on Fermat hypersurfaces, and maximal Picard rank of the Fermat sextic hypersurfaces in $\mathbb{P}^3$ (Beauville, 2014).

Lecture 3: Mod $p > 0$ reduction of transcendental motives, an arithmetic view

In the third lecture I will give an outline of a new (unpublished) method of proving that the transcendental motive of a smooth projective surface is integrally indecomposable. The main idea is that integral indecomposability of $M^2_{\mathrm{tr}}(X)$ can be always lifted from the closed fibre to the generic one, in a smooth projective family of surfaces over the spectrum of a DVR, and this allows us to apply the reduction mod $p > 0$ idea, joint with another result saying that some Fermat hypersurfaces in some positive characteristics have integrally indecomposable $M^2_{\mathrm{tr}}(X)$. We hope that this approach gives a complete proof that the transcendental motive of any hypersurface in $\mathbb{P}^3$ is integrally indecomposable.